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Pricing and the Distribution of Money
Holdings in a Search Economy

by

Peter Diamond and Joel Yellin

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massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

Department of Economics
Massachusetts Institute of Technology
50 Memorial Drive
Cambridge, MA 02139

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Peter Diamond, Massachusetts Institute of Technology

Joel Yellin, University of California, Santa Cruz

Introduction

Central to traditional microeconomics is the Walrasian auctioneer who coordinates trades and sets prices. Continuing our earlier work (Diamond and Yellin (1985a), referred to below as A), we present a two-market, general equilibrium model in which the auctioneer plays no role in the retail market. Two non-Walrasian features of observed economic behavior are captured by our model. First, most retail sellers face downward sloping demand curves. We introduce downward sloping demand by using a model based on search. We eschew the more traditional approach of

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monopolistic competition (e.g., Hart (1982)). Second, retail buyers are constrained by access to purchasing power. We therefore impose a finance constraint (cf. Clower (1967), Kohn (1981)), restricting purchasing power to concurrent money holdings. We do not consider credit. This work should be considered an interim step in the development of models with richer, more realistic structure in which credit plays its appropriate role.

Our assumptions are strong. They enable us to solve the model explicitly, both in the steady state and in the immediate aftermath of an instantaneous, equal per capita infusion of money. The model economy has a unique steady state equilibrium in which the greater the efficiency of the search process, the higher the absolute levels of the retail price and the wage rate, and the lower the ratio of price to wage. As the speed of the search process increases, the steady state inventories and relative price converge to values that characterize the Walrasian steady state. In partial equilibrium versions of the same model, there is a discontinuity in the equilibrium price as the cost of search goes to zero (Diamond (1971)). The critical feature of this model that eliminates the partial equilibrium discontinuity is a general equilibrium link between the distribution of money holdings and the speed of the search process.

Similar comparative statics results were obtained in the pure barter search model analyzed in A. To introduce money, we have given the present model a richer structure: we distinguish two groups of agents -- workers and capitalists; there is production; there is a circular flow of money; and capitalists maximize the present discounted value of utility of their own-consumption streams.

Our work is similar to that of Lucas (1980). However, he assumes

Walrasian retail markets and random preferences, while we use a search technology and assume random purchasing opportunities. Moreover, our techniques differ. Lucas' analysis is based upon a discrete-time formulation that leads him to use measure-theoretic tools. Our continuous-time formulation enables use of the simpler tools of classical analysis.

With these tools, we take a first step in dynamic analysis. We compute the instantaneous effects on price, wage, and transaction rate of a one-time, equal per capita monetary infusion. We show that the infusion increases the transactions rate, prices, real wages and wages. The relationship between the instantaneous post-infusion price and its asymptotic value depends on the ratio of the capitalists' utility discount rate to the rate of retail search and on the size of the infusion. A high discount rate and a large infusion cause the instantaneous post-infusion price to exceed its asymptotic level. Small infusions and relatively high search rates lead to the opposite result. The instantaneous post-infusion wage exceeds its asymptotic value for a larger set of parameter values. While these conclusions depend on our particular specification of the mechanisms that determine the elasticity of retail demand, the model shows that it is feasible to use a fully specified micro-based approach in determining prices. The success of the present calculations underlines the importance of such an approach.

1. Description of Model

Our model economy is pictured in Figure 1. It is peopled by two classes of agents, workers and capitalists. Each worker receives a flow of labor endowment at unit rate. He sells his labor services to capitalists

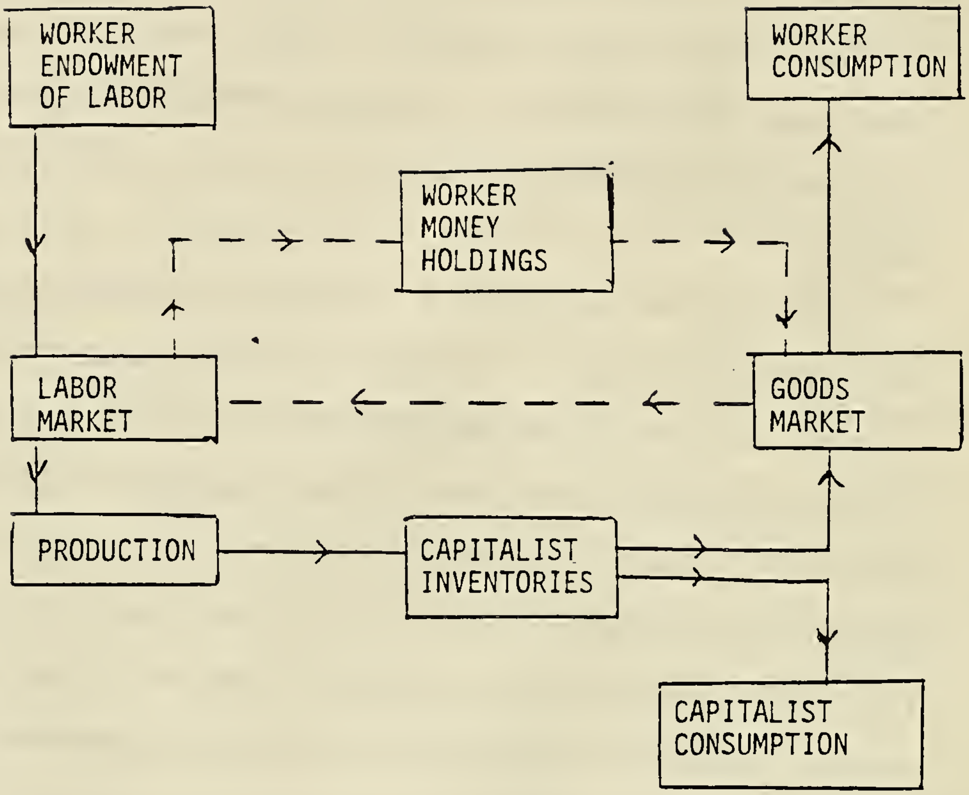

Figure 1.

Schematic flow diagram depicting monetary model analyzed in text. Labor market is Walrasian. Goods market is mediated by random search technology that allows transactions at Poisson intervals. In each transaction, a worker purchases a commodity bundle of unit size. Solid arrows are physical flows; dashed arrows indicate money flows.

on a labor market mediated by a Walrasian auctioneer.* Labor services are continuously compensated at a market clearing rate w. A worker thus receives money wages at the rate w, and labor services flow to capitalists at a rate proportional to the total number of workers in the economy. We shall assume there are sufficiently large numbers of workers and capitalists that each group may be considered to form a continuum. We assume N workers per capitalist; we normalize the continuum of capitalists to 1. We assume further that there is a fixed aggregate quantity of money.

The economic environment of capitalists is as follows. Capitalists hire workers on the labor market and employ them to produce goods continuously. Labor is the only productive factor. A unit of labor produces a unit bundle of goods with no delay. We assume that all capitalists are identically situated, that there are no inventory carrying costs, and that capitalists have no retail price reputations.

Capitalists use their inventories in two ways. First, they consume them continuously at a per capita rate, c, set by lifetime utility optimization rules described below. Second, they offer them for sale in the retail market at a price, p, per unit bundle. This price is also set in the lifetime optimization process. Retail revenues are disposed of immediately** upon receipt and are used solely to purchase labor services.*** We assume each retail transaction results in the purchase of a single bundle of goods.

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*There is no difficulty in principle in making the entire model non-Walrasian by introducing search and long-term contracts in the labor market.

**In a sequel (1985b), we explore the consequences of capitalist money holdings.

***Given a constant wage, capitalists prefer to hire additional labor for production rather than to hold money that has no yield.

The structure of the retail market is as follows. We assume retail trade is mediated by a random search technology that enables workers to search for and locate consumption opportunities. Such opportunities come at times set by a Poisson process with an endogenous arrival rate. Each consumption opportunity entails the possibility of purchasing a unit bundle of the consumer good at a price p that is set by capitalists. There is no credit, and to make a purchase a worker must hold money in amount at least equal to p. We picture each bundle of inventory as available for purchase at a separate retail outlet.* A successful purchase by a worker is followed by consumption of the proceeds and results in a fixed gain of utility. With constant prices and a positive utility discount rate workers want to make every purchase they can.

Two accounting identities are fundamental to the model. First, let T be the rate at which retail transactions occur, per capitalist in the economy. Then, since capitalists spend the proceeds of retail sales instantly, clearing of the labor market implies the total wage bill of each capitalist is equal to his retail revenue:

$$wN = pT. \tag{1.1}$$

The second identity describes the growth of capitalists' inventories. Let the inventory of commodity bundles per capitalist be x. Inventories grow at the rate pT/w, per capitalist, due to production by labor hired with the (cash) proceeds of retail sales. Inventories shrink at the rate T

*In a more realistic, but also more complicated formulation, one would consider the distribution of inventories over the set of retail outlets of a single capitalist.

due to retail sales and also at the rate c, as the result of capitalists' consumption of their own inventory. Therefore, we have the accounting relation for per capitalist flows,

$$dx/dt = -c - T + pT/w. \tag{1.2}$$

While workers experience uncertainty in their purchasing opportunities, we assume a sufficient number of workers per capitalist for the capitalist's economic environment to be determinate.

The workers' choice problem is as follows. We assume that workers' preferences are described by the expected present discounted value of utility of consumption, with a strictly positive discount rate. There is no credit, and money holdings must therefore always be non-negative. Since we consider only uniform price equilibria, no worker ever encounters a price sufficiently high to give him an incentive to wait for a better price. Given these simple time preferences and the lack of credit, in a constant-price equilibrium workers make every possible purchase. In particular, purchase opportunities are taken whenever a worker's money holding exceeds the retail price p.* Thus worker money holdings increase continuously at rate w and decrease stochastically in jumps of size p.

Our search model of the retail market leads to specification of the retail transaction rate, T, in terms of the money distribution and the total inventory. Let F(m) be the distribution of workers' money holdings. Then the fraction of workers with money sufficient to consummate a purchase is 1-F(p). In the simplest random search,** the search technology enables

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*Out of steady-state, a worker's choice problem depends on his perception of future prices.

**We generalize the search technology in Section 5.

the random pairing of a worker with a single commodity bundle. The transaction rate per capitalist therefore is

$$T = hN[1-F(p)]x, \tag{1.3}$$

the product of the search intensity, h, the number of workers, $N[1-F(p)]$, actively searching for consumption opportunities, and the total number of commodity bundles, x, in commerce.

In this picture in equilibrium each worker experiences an arrival rate, b, of consumption opportunites equal to the retail transactions rate per worker actively engaged in search. Using (1.3), we therefore write

$$b = \frac{T}{N[1-F(p)]} = hx. \tag{1.4}$$

That is, each worker has the flow probability h of meeting each unit bundle of goods available for sale.

Thus, workers' money holdings obey a stochastic process. They increase continuously at the wage rate, w. They decrease in jumps equal to the retail price, p, subject to the condition that no individual's money holdings can become negative. Jump times are set by a Poisson process whose arrival rate is proportional to the total inventory in the market and the efficiency of the search technology.

We may formalize this stochastic process as follows. Let f(m) be the asymptotic probability density of a worker's money holdings. In a steady state, flows into and out of any level of money holdings are equal. On the assumptions above, workers' equilibrium money holdings are determined by

$$wf'(m) = hx[f(m+p) - \underline{\theta}(m-p)f(m)]. \tag{1.5}$$

The left-hand side of (1.5) represents the inflow of money wages, and the

right-hand side represents the net outflow due to retail transactions.* The solution of the difference-differential equation (1.5) is given in Section 2.

It remains to specify the choice problem for capitalists. We view capitalists as consuming continuously at a rate c out of their own inventories, maximizing the present discounted value of the utility of their own-consumption streams, with a positive utility discount rate r. Then, with the accounting relations (1.1) and (1.2) in mind, each capitalist optimizes by monitoring the time paths of price and inventory. In optimizing, capitalists are aware that the equilibrium distribution of money holdings satisfies (1.5). However, they assume that their own price-setting behavior does not affect workers' money holdings. This is the standard competitive assumption, and is appropriate with many capitalists. Formally, the capitalists' choice problem is therefore

$$\max V[x(t), p(t)] = \int_0^{\infty} e^{-rt} u[c(t)]\, dt = \int_0^{\infty} e^{-rt} u[-dx/dt - T + pT/w]dt, \quad (1.6)$$

where we take u(c) to be monotone increasing and concave, with $u'(0)=\infty$. The Euler conditions that follow from (1.6) are discussed in Section 3.

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*The symbol $\theta(x)$ denotes the Heaviside (unit step) function, which is 1 for positive x and 0 otherwise.

2. Equilibrium Money Holdings

The methods developed in A enable us to solve (1.5) for the equilibrium distribution of workers' money holdings and to assert that the solution thus obtained is unique. We note first that it is a property of the model described in Section 1 that workers with sufficient funds are able to spend more rapidly than money is received. Formally, in equilibrium we have the inequality between endogenous quantities

$$w \leq bp. \tag{2.1}$$

The condition (2.1) is required for logical consistency. If this inequality does not hold, there is unlimited growth of the stock of money, and this contradicts our assumption that the money stock is fixed.

Given (2.1), the arguments in A tell us that the asymptotic distribution of money holdings is uniquely given by

$$F(m) = (b/wk)[1-F(p)][1-\exp(km)+km]; \quad m\leq p \tag{2.2a}$$

$$F(m) = 1-[1-F(p)]\exp[k(m-p)], \quad m\geq p \tag{2.2b}$$

where k is the unique real negative root of

$$wk = b(e^{kp}-1), \tag{2.3a}$$

and

$$1-F(p) = w/bp = \frac{e^{kp}-1}{kp} . \tag{2.3b}$$

By direct substitution, one verifies that (2.2) solves (1.5). As pointed out above, we assume there is a continuum of individual workers. Therefore, the aggregate distribution of money holdings incorporates no statistical uncertainties, and the asymptotic probability distribution for a single worker engaged in the stochastic trading process is also the asymptotic equilibrium distribution of money holdings, (2.2), over all workers in the economy.

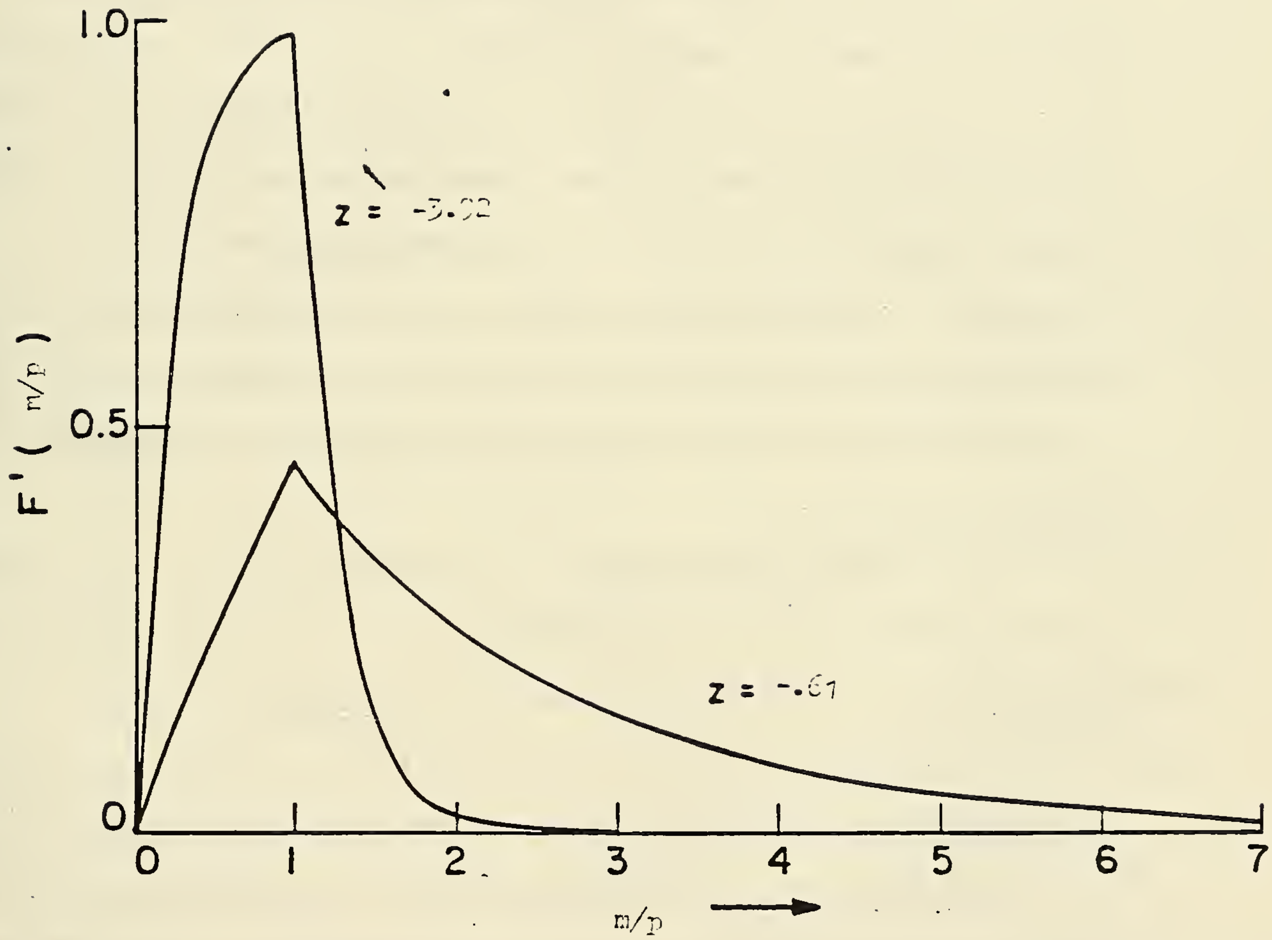

Figure 2

Probability density of money holdings defined by equation (2.2) of text. Note the flattening and extension of the right-hand tail as z increases. For the two densities shown, 1-F(p) is equal to 1/4 and 3/4.

We gain insight into the economics of the stochastic process governing workers' money holdings by relating the elasticity of demand in equilibrium to the parameters of the money distribution. Basic to our search model is the assumption that workers sample suppliers one at a time. Thus, in a uniform price equilibrium, it is natural to insist that workers are willing to pay, if able, at least a little more than the going price for the opportunity to make an immediate purchase. Since workers cannot spend more than they possess, in the neighborhood of the equilibrium price the demand curve for each capitalist is given by the distribution of workers' money holdings times the number of buyers that come in contact with his inventory. In particular, the demand curve in equilibrium, at a price level m near p, is proportional to the fraction of workers, $1-F(m)$, holding at least the money level m. We may therefore define the elasticity of aggregate demand as

$$z = d\log[1-F(m)]/d\log m \Big|_{m=p} . \tag{2.4}$$

From (2.2),

$$z = pk. \tag{2.5}$$

Since monopolists do not set prices when demand is inelastic, only values of z less than -1 are of interest here. Apart from the scale factor p, the family of distributions (2.2) is a one parameter family of distributions. We can use z as this parameter. In Figure 2 we show the density for two values of z.

If we compute the mean money holding per worker, $\bar{m}$, implied by (2.2) we find

$$\bar{m}/p = \tfrac{1}{2} - 1/kp = \tfrac{1}{2} - z^{-1}. \tag{2.6}$$

As already stated, the number of workers and the nominal money stock are assumed to be fixed exogenous quantities, and workers hold all money in the economy, since capitalists spend the proceeds of retail sales without delay. Therefore, $\bar{m}$ is a constant parameter of our model, and (2.6) implies the elasticity of aggregate demand in steady state decreases monotonically as the retail price increases.

3. Capitalist Behavior

The optimization problem (1.6) for capitalists leads to two first-order Euler equations, one arising from the variation with respect to the time path of price, the other arising from the variation with respect to the time path of inventory. On varying the price and substituting for T from (1.3), we find that optimal price-setting is ensured by

$$1-w/p = [1-F(p)][pF'(p)]^{-1} = -1/z. \tag{3.1}$$

Here, on the argument above, the elasticity z is taken to be less than -1. Since capitalists have no price reputations and instantly convert the revenue from sales into additional inventory, pricing behavior maximizes instantaneous real profits. In (3.1), the notation $F'(p)$ denotes the density dF/dm evaluated at $m=p$. Consistent with our competitive assumption, we do not differentiate $F(m)$ with respect to p, and the structural parameters of the money distribution therefore do not enter into the first-order maximization condition.

Equation (3.1) states that capitalists set the retail price, p, to maximize $(p/w-1)T$, the inflow (gross of consumption) of goods. The right-hand side of (3.1) is positive, and the real wage, w/p, is therefore less than the (unit) marginal product of labor. Observe also that the real wage is a decreasing function of the elasticity, z.

In addition to the first-order price setting condition (3.1), there is a second Euler condition analogous, in its asymptotic form, to the modified Golden Rule in optimal growth problems. In particular, when the system reaches equilibrium, the optimal inventory level is such that the discount rate equals the own marginal product of inventories. Formally, on varying x(t), (1.6) yields the equilibrium condition

$$r = h(p/w-1)N[1-F(p)]. \tag{3.2}$$

With each unit of inventory separately available for sale and many capitalists, the average return on inventories equals the marginal return on inventories. Thus, in steady state, the consumption of capitalists equals the utility discount rate times the stock of inventories, as can be confirmed from the equations below.

4. Comparative Statics

Using the Euler condition (3.1), the asymptotic Euler condition (3.2), the equations for the holdings of money (2.3a, 2.5, 2.6), the cash flow identity (1.1), and the random search assumption (1.4), we can analyze the comparative statics of steady states given different parameter choices. In particular, we may relate the various quantities of economic interest to the elasticity, z. The same relations allow us to express z as an implicit function of the ratio, r/h, of the discount rate to the speed of search. Formally, we have

Theorem 4.1. The model described above has a unique uniform * price

*We have not considered whether there also exist equilibria with different prices in different transactions.

equilibrium in which the real wage, w/p, and the real money supply, m/p, are fixed by the ratio, r/h, of capitalists' discount rate to the speed of retail search.

Proof. On rewriting (3.1), the real wage may be expressed as

$$w/p = 1 + 1/z. \tag{4.1}$$

On eliminating the elasticity z from (4.1) and the money supply equation (2.6), we have the simple sum rule for the real wage and real money supply

$$w/p + \bar{m}/p = 3/2. \tag{4.2}$$

This equation represents the flow equilibrium in price setting. We obtain a second relationship between real wage and real money supply from the equation for stock equilibrium of inventory holdings, (3.2). From (2.3b), (2.5), and (2.6), aggregate demand evaluated at the equilibrium price may be expressed as

$$1-F(p) = (e^{z}-1)/z = (e^{2/(1-2\bar{m}/p)}-1)((1/2)-(\bar{m}/p)). \tag{4.3}$$

Combining (3.2) and (4.3) we have

$$((1/2)-(\bar{m}/p))(e^{2/(1-2(\bar{m}/p))}-1) = (r/hN)((w/p)/(1-(w/p)). \tag{4.4}$$

One verifies that in (4.4), w/p is monotone increasing in $\bar{m}/p$ in the relevant interval $0 \leq w/p \leq 1$. On the other hand, the simple linear relationship (4.2) implies w/p is decreasing in $\bar{m}/p$. Therefore (4.2) and (4.4) have a unique solution. Q.E.D.

We plot (4.2) and (4.4) in Figure 3 and display the equilibrium solution.

The model yields the following further equilibrium relations that will be useful below. From (2.6) and (4.1), we may express the wage and retail

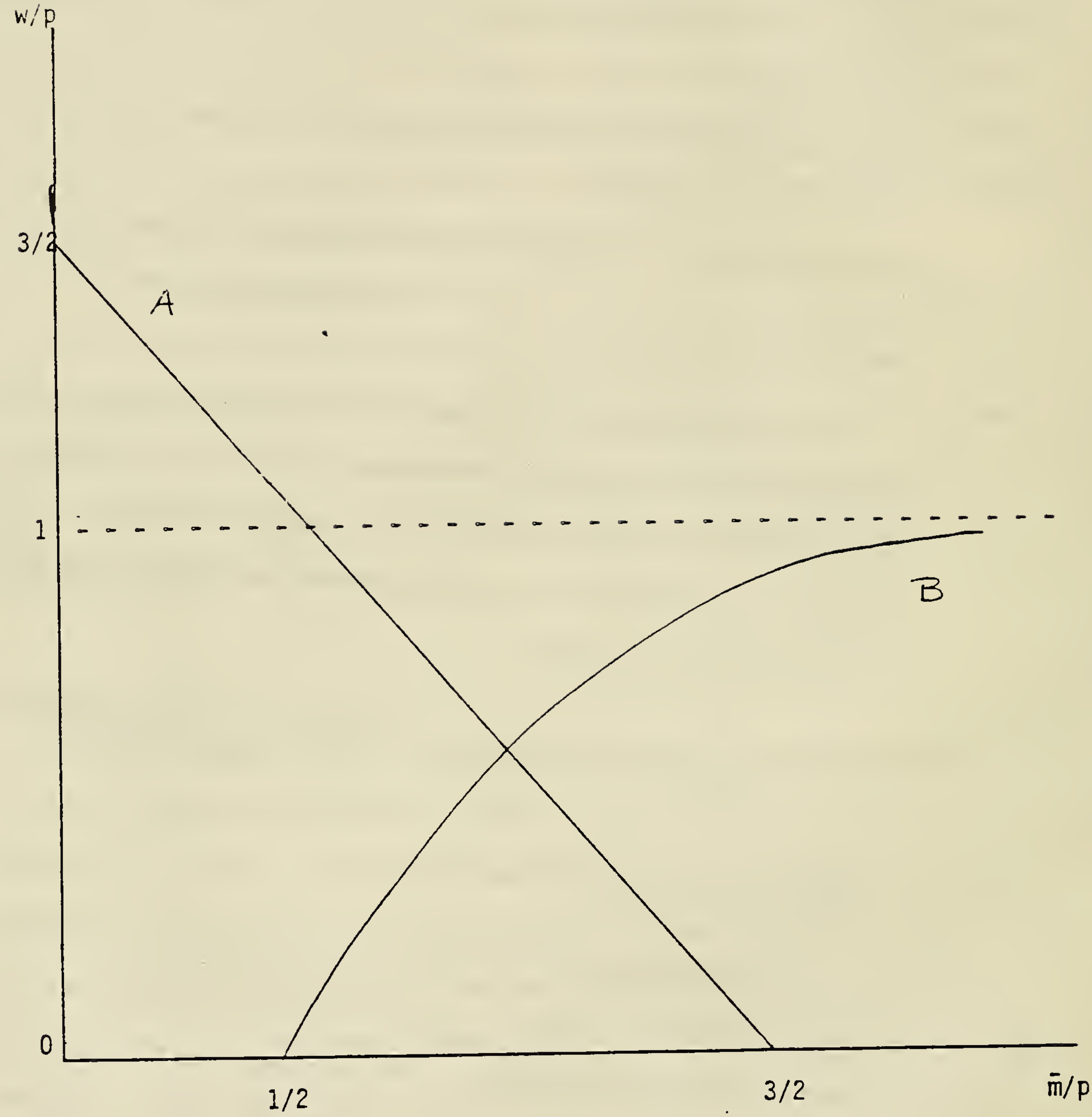

Figure 3

Behavior of real wage versus real money supply, as given by equations (4.2) (curve A) and (4.4) (curve B) of text, showing unique equilibrium.

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price in terms of the elasticity. We have

$$w = 2\bar{m}(1+z)/(z-2); \tag{4.5}$$

$$p = 2\bar{m}(1-2/z)^{-1}. \tag{4.6}$$

From (3.2), (1.4), (3.1), and (1.1), the equilibrium inventory is

$$x = -N(rz)^{-1}. \tag{4.7}$$

Using (2.5) and (4.1), the characteristic equation (2.3a) allows us to relate the Poisson arrival rate, b, to the elasticity. We have

$$b = (1+z)(e^{z}-1)^{-1}. \tag{4.8}$$

From (4.8), b is a decreasing function of z, for (feasible) values of z less than -1.

Finally, the simple random search assumption (1.4) allows us to express the elasticity -- and therefore the price and wage -- in terms of the ratio r/h. Using (3.2), (4.1), and (4.3) we have

$$r/h = N(1-e^{z})z^{-1}(1+z)^{-1}. \tag{4.9}$$

We show (4.9) in Figure 4. Note that z and h are inversely related. In particular, as h runs from 0 to values indefinitely large, -z runs from 1 to values indefinitely large. In Figure 5, we use (4.5), (4.9), and (4.6) to show the behavior of the retail price and wage as a function of r/h. As -z rises from 1, p increases from $2\bar{m}/3$ to $2\bar{m}$, w runs from 0 to $2\bar{m}$, p/w runs from $\pm\infty$ to 1. The Walrasian equilibrium has p=w. As shown in Figure 5, the wage/price ratio converges to the Walrasian limit as the speed of search increases. In the Walrasian limit, purchases are instantly made once adequate money holdings are accumulated. The nominal Walrasian price is therefore $2\bar{m}$, and in the Walrasian limit workers' money holdings are distributed uniformly on the interval 0 to p. Average money holdings are then half the retail price. With an infinite search speed there is no

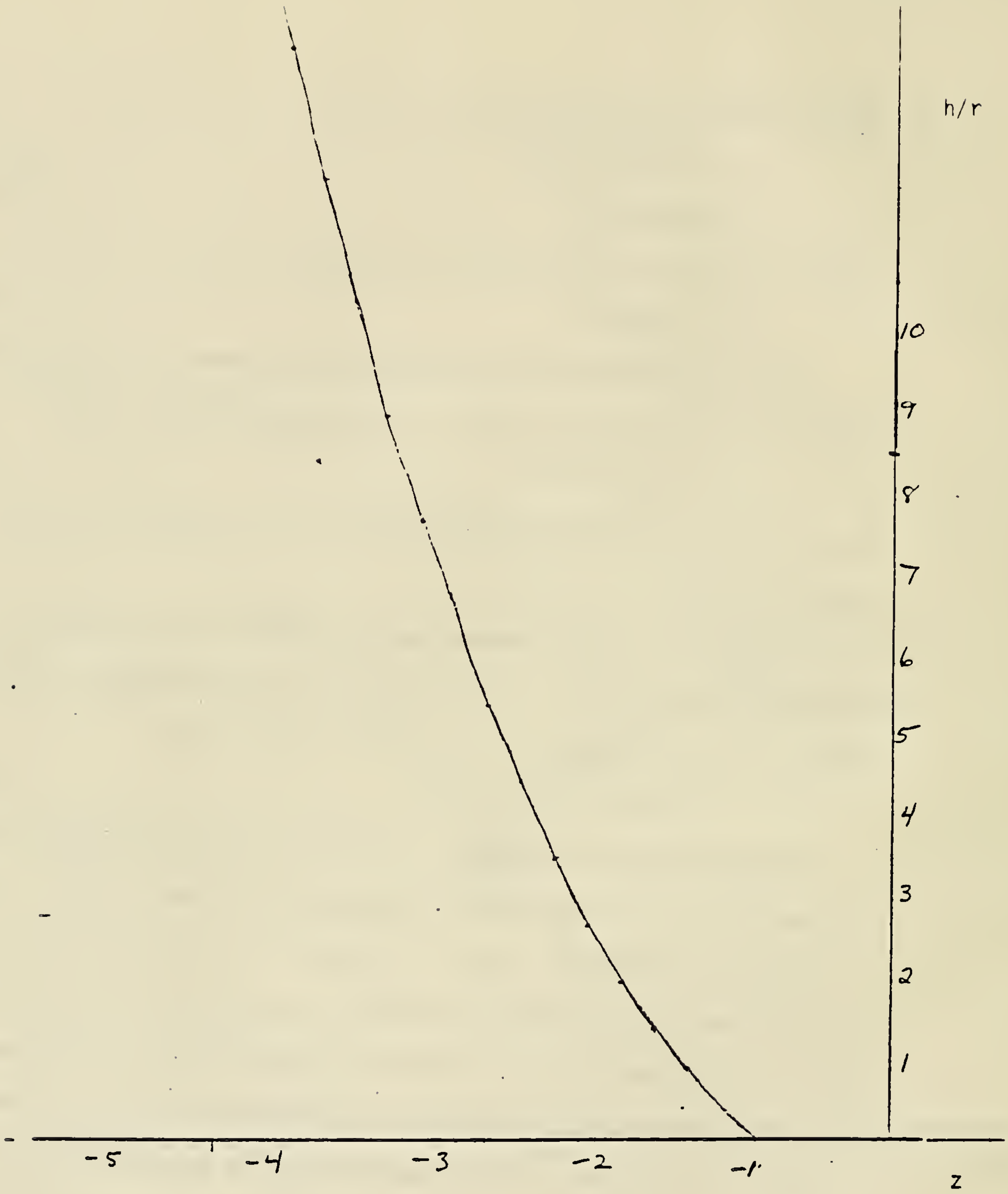

Figure 4

Equilibrium demand elasticity as a function of ratio of search speed to discount rate of capitalists.

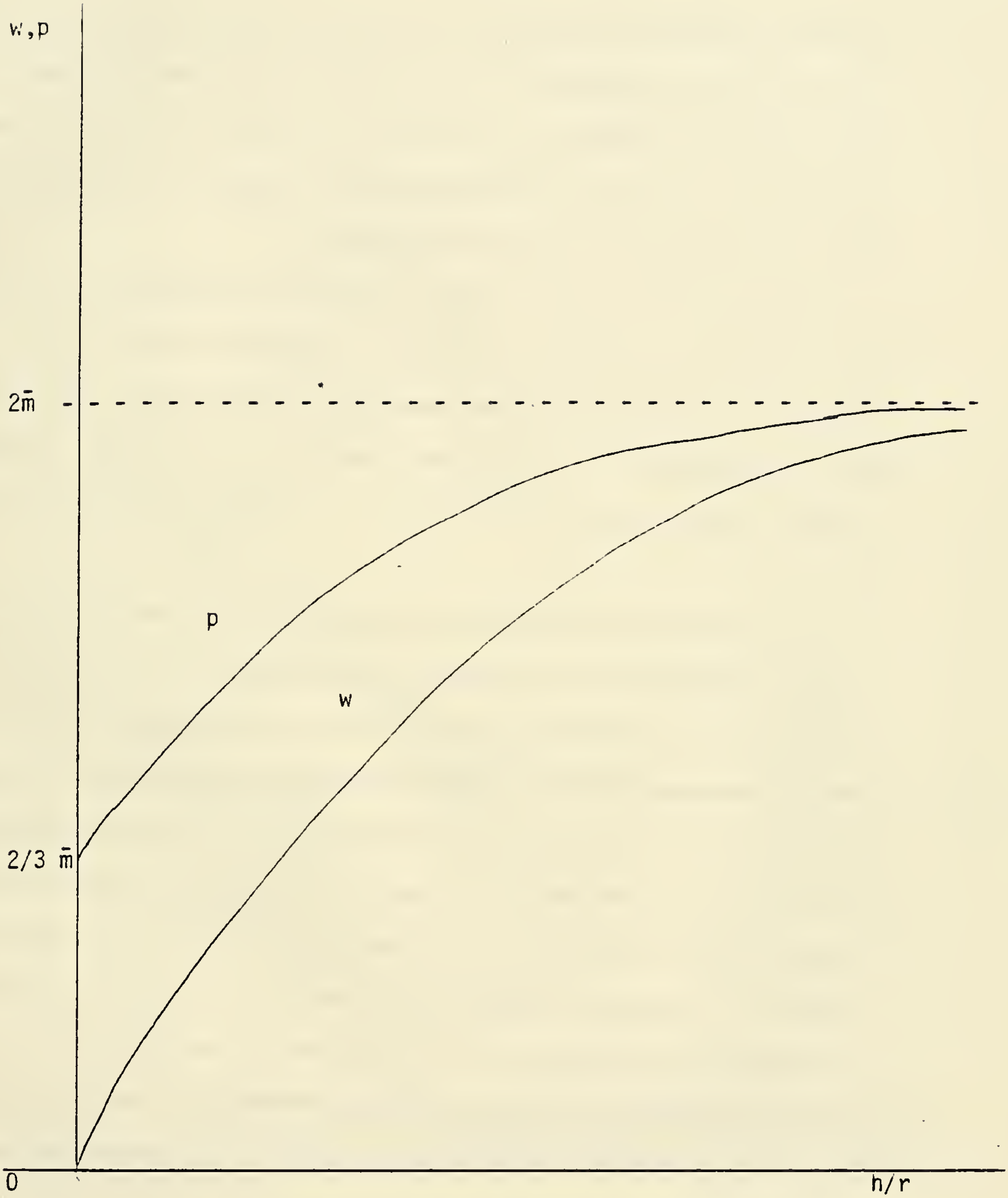

Figure 5

Behavior of retail price and wage as functions of ratio of search speed to discount rate of capitalists.

lag from production to sale and no monopoly power. Thus there is no markup over the cost of production. At the other extreme, when h becomes small the elasticity of demand is -1, and costs are zero. This infinite markup translates into a specific nominal price by equating the implied average money holdings with the nominal money stock.

Figure 6 sets out the relation of the total inventory, x, to h using (4.7) and (4.9). As h rises from zero, x decreases from N/r to 0. An increase in r shifts this curve downward. Differentiating (4.9), we see that rz decreases in z. Thus x decreases in z, and so in r. Therefore, an increase in the discount rate lowers p, w, and x, but increases the markup p/w.

The smooth approach to Walrasian equilibrium as the speed of search increases stands in sharp contrast to the results of partial equilibrium analyses of the same search model. In the partial equilibrium approach, the equilibrium allocation does not converge to the Walrasian solution as search costs go to zero, since the demand curve is taken as exogenous. This implies that the price is independent of the search technology as long as search costs are positive. An understanding of the limiting Walrasian behavior of the present model follows from the observation (cf. (4.9)) that the elasticity of demand varies with the search technology, growing without limit as the speed of the search process increases. Thus the markup factor p/w goes to 1 in the Walrasian limit, as implied by Figure 5. The endogeneity of the distribution of consumer limit prices distinguishes this general equilibrium model from its partial equilibrium counterpart (see Diamond (1971)).

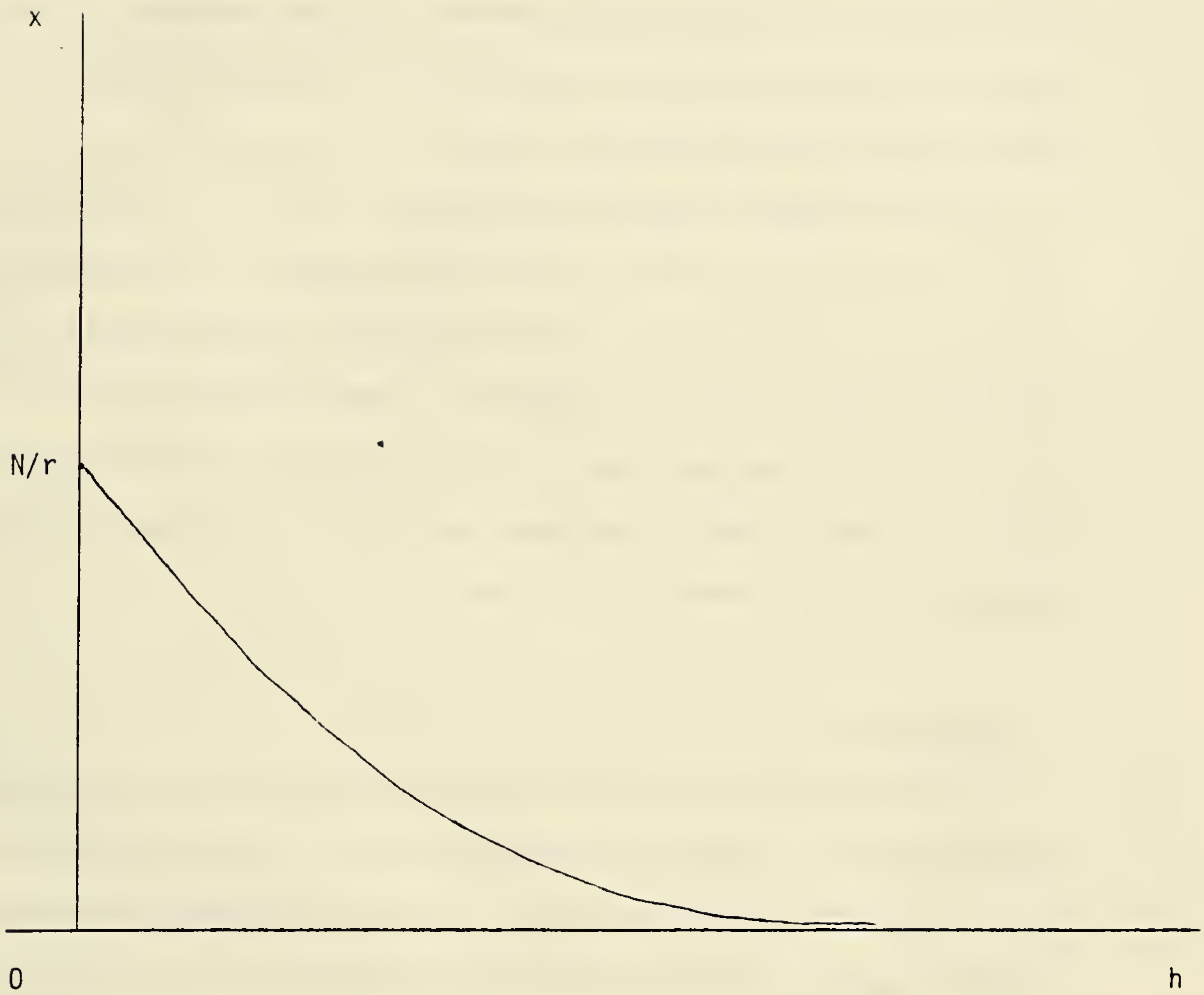

Figure 6

Capitalist inventory holdings in equilibrium as function of search intensity in goods market. As in Figure 5, note the approach to the Walrasian equilibrium with zero inventory holdings as the efficiency of search increases.

Observe also that the instantaneous velocity of money -- the ratio of the flow of nominal retail transactions to the stock of money holdings -- here takes the simple form (cf. (1.1))

$$v = pT/\bar{m}N = w/\bar{m} = 2(1+z)/(z-2). \tag{4.10}$$

Thus, the velocity of money in equilibrium behaves as the wage rate, going from 0 to 2 as the speed of search grows without limit from 0. In the Walrasian limit it takes one period to accumulate money sufficient to purchase a unit bundle of goods. Thus, average money holdings are half the price of a unit bundle in the Walrasian limit, and the corresponding Walrasian money velocity is 2.

5. Generalities

The qualitative relationships described above carry over to more general models. When search intensity is not a choice variable, and when there is no intertemporally varying structure to search, the transactions rate, T, depends solely on the number of would-be purchasers N[1-F(p)], and on the mean stock of inventories per capitalist, x. Then the generalization of the second equality in (1.4) is

$$T = hH(N[1-F(p)],x), \tag{5.1}$$

where h is a scale factor suitable for comparative static analysis, and H has well behaved isoquants. Plausibly, H is increasing in each argument and, in the absence of congestion, has returns to scale between 1 and 2. Nevertheless, given the continuum of capitalists, there is no stochastic uncertainty, and each capitalist still perceives a linear relationship between sales and inventory stocks, since no individual's activities are significant enough to affect the ratio of sales to inventory. Given (5.1),

one can show that the model has a unique solution for all h, assuming a well-behaved function H. Moreover, the monotone relations between p, w, p/w, x, and z described in Section 4 continue to hold. The elasticity z now depends on r and h independently, however, rather than only on the ratio h/r.

6. One-Time Monetary Infusion

We now consider the instantaneous effects of an unexpected one-time money infusion equally distributed in an amount m' per worker. We assume that perceptions of future prices have no effect on workers' consumption decisions. Thus, in the aftermath of the infusion, workers continue to accept every consumption opportunity. A sufficient condition that this purchasing behavior is optimal is that retail prices are rising. Or, if they are falling, universal acceptance is still the rule if prices decrease sufficiently slowly. We do not attempt to define "sufficiently slowly." Nor do we analyze full dynamic price trajectories. We restrict ourselves to comparisons of the initial steady state price, the instantaneous post-infusion price (assuming all possible purchases are made) and the asymptotic steady state. Our analysis is clearly relevant for sufficiently small monetary infusions. We conjecture that it is also relevant for large infusions provided the instantaneous post-infusion price is below the asymptotic price.

Given an equal per capita infusion, we expect immediate discontinuous jumps in the retail price and wage, p and w. Similarly, there is a jump in the flow of transactions, T. On the other hand, the physical stock of goods in inventories responds in finite time to the infusion; in particular, in the post-infusion instant, capitalists' inventories x remain

unchanged. Since workers continue to accept every consumption opportunity, the post-infusion distribution of consumer limit prices still coincides with the distribution of money holdings. The shape of this distribution is unaffected by the infusion, but the entire distribution shifts to the right by m'. We may therefore write the post-infusion distribution as

$$F_I(s) = F(s-m'). \tag{6.1}$$

We may draw qualitative conclusions about the consequences of the infusion from (6.1). By shifting the distribution to the right, we lower the elasticity of demand at the peak of the density, which occurs at a price equal to the previous equilibrium price plus the infusion. Thus, price does not increase by the full amount of the monetary infusion. In comparison to the ultimate new asymptotic equilibrium, the instantaneous post-infusion tails of the density are shorter. When the infusion is small or the search process is sufficiently rapid, firms increase price to a level less than the ultimate asymptotic value. But when the infusion is sufficiently large and the search process is sufficiently slow, overshooting will occur. When the price overshoots, the infusion of money temporarily lowers the real money stock. Nevertheless, the transactions rate rises. The instantaneous wage exceeds the asymptotic wage for a larger set of parameter values.

To confirm these qualitative observations, we derive a pair of equations that enable us to express the post-infusion price and wage, p' and w', in terms of the infusion, m' ($m' \geq 0$), and the pre-infusion parameters. First, we observe from (1.1) that clearing of the labor market and instantaneous flows of money between markets imply the accounting relation pT = Nw is unaffected by the infusion. Recalling that the

inventory x is also unchanged, we may use the random pairing model (1.3) for the sales rate T to express the post-infusion markup as

$$p'/w' = (p/w)\,[1-F(p)]/[1-F_I(p')]. \tag{6.2}$$

A second relation follows from substituting the shifted distribution (6.1) into the price-setting equation (3.1), obtaining

$$1-w'/p' = \frac{1-F(p'-m')}{p'f(p'-m')}. \tag{6.3}$$

On the other hand, we may write (6.2) as

$$1-w'/p' = 1 - \frac{1-F(p'-m')}{(p/w)[1-F(p)]}. \tag{6.4}$$

By inspection of (6.3) and (6.4), a simultaneous solution of these equations is possible only if the quantity p'-m' is positive, consistent with the common-sense result that a firm will not set a price that is lower than the minimum amount of money held by consumers. Moreover, as we shall confirm explicitly below, the properties of the money distribution F(s) are such that the right-hand side of (6.3), considered as a function of p', <u>decreases</u> monotonically from infinity at p'=m' to 0 when p' is indefinitely large. On the other hand, the right-hand side of the market clearing relation (6.4) <u>increases</u> monotonically over the same range. Therefore, there is a unique post-infusion price p'. Formally, we have:

<u>Theorem 6.1</u> There is a unique post-infusion price, p', such that $0 \leq p'-m' \leq p$.

<u>Proof.</u> (<u>1</u>) As pointed out above, values of $p' \leq m'$ may be excluded by observing that for negative arguments, F=f=0, and (6.3) and (6.4) are then inconsistent.

($\underline{2}$) To exclude the region $p'\geq p+m'$ we use (2.2b) to rewrite (6.3) and (6.4) as

$$1 - w'/p' = -p/zp'; \tag{6.3'}$$

$$1 - w'/p' = 1 - (w/p)e^{k(p'-p-m')}. \quad p'\geq p+m' \tag{6.4'}$$

The result follows immediately from two observations. First, at $p'=p+m'$, the right-hand side of (6.3') is less than the right-hand side of (6.4'). Second, the right-hand side of (6.3') is monotone decreasing in p', while the right-hand side of (6.4') is monotone increasing. Therefore, values of p' greater than $p+m'$ are excluded, and we conclude that a firm will not set a price p' that exceeds the pre-infusion price by more than the size of the infusion.

($\underline{3}$) Consider the remaining region $m'\leq p'\leq p+m'$. In this region, equation (2.2a) implies that $f(p'-m')$ is monotone increasing and $1-F(p'-m')$ is monotone decreasing. Thus, the right-hand side of (6.3) is monotone decreasing, while the right-hand side of (6.4) is monotone increasing, as p' runs from m' to $p+m'$. It follows immediately that region ($\underline{3}$) contains a unique solution p'. Q.E.D.

Since the price rises by less than the monetary infusion, the fraction of workers able to purchase increases. Thus we have

Corollary The post-infusion transaction rate exceeds the pre-infusion steady state transaction rate.

Theorem 6.2.

For all feasible values of m' and z, the instantaneous post-infusion price p' exceeds the pre-infusion price p.

Proof.

If we equate (6.3) and (6.4), we find that the unique equilibrium price p' satisfies

$$\frac{p'}{p} = \frac{1 - F(pt)}{pf(pt)[1-b(1-F(pt))]} , \tag{6.5}$$

where we have defined the quantity

$$t=(p'-m')/p \quad . \tag{6.6}$$

and have used (2.3b) in the form $1/b=(p/w)[1-F(p)]$. From Theorem 6.1, we know that $0\leq t\leq 1$.

From (2.2a), we write (6.5) as

$$t + \frac{m'}{p} = \frac{p'}{p} = \left(\frac{1-e^{z}}{1-e^{zt}}\right)\left(1 + z\left(\frac{e^{z}-z-e^{zt}+zt}{1-z-e^{zt}+zt}\right)\right)^{-1} \equiv A(t;z) \tag{6.7}$$

For $0\leq t\leq 1$, $z\leq -1$, the right-hand side of (6.7) is monotone decreasing in t, going from $+\infty$ to 1 as t goes from 0 to 1. Thus $p'/p\geq 1$ at the value of t that satisfies (6.7). Q.E.D.

We exhibit the price jump, p'/p, as a function of the ratio r/h (not h/r as in earlier figures)in Figure 7. Observe the singular behavior of price in the Walrasian limit $h=\infty$

Theorem 6.3. For all feasible values of m' and z, the instantaneous post-infusion wage w' exceeds the pre-infusion wage w.

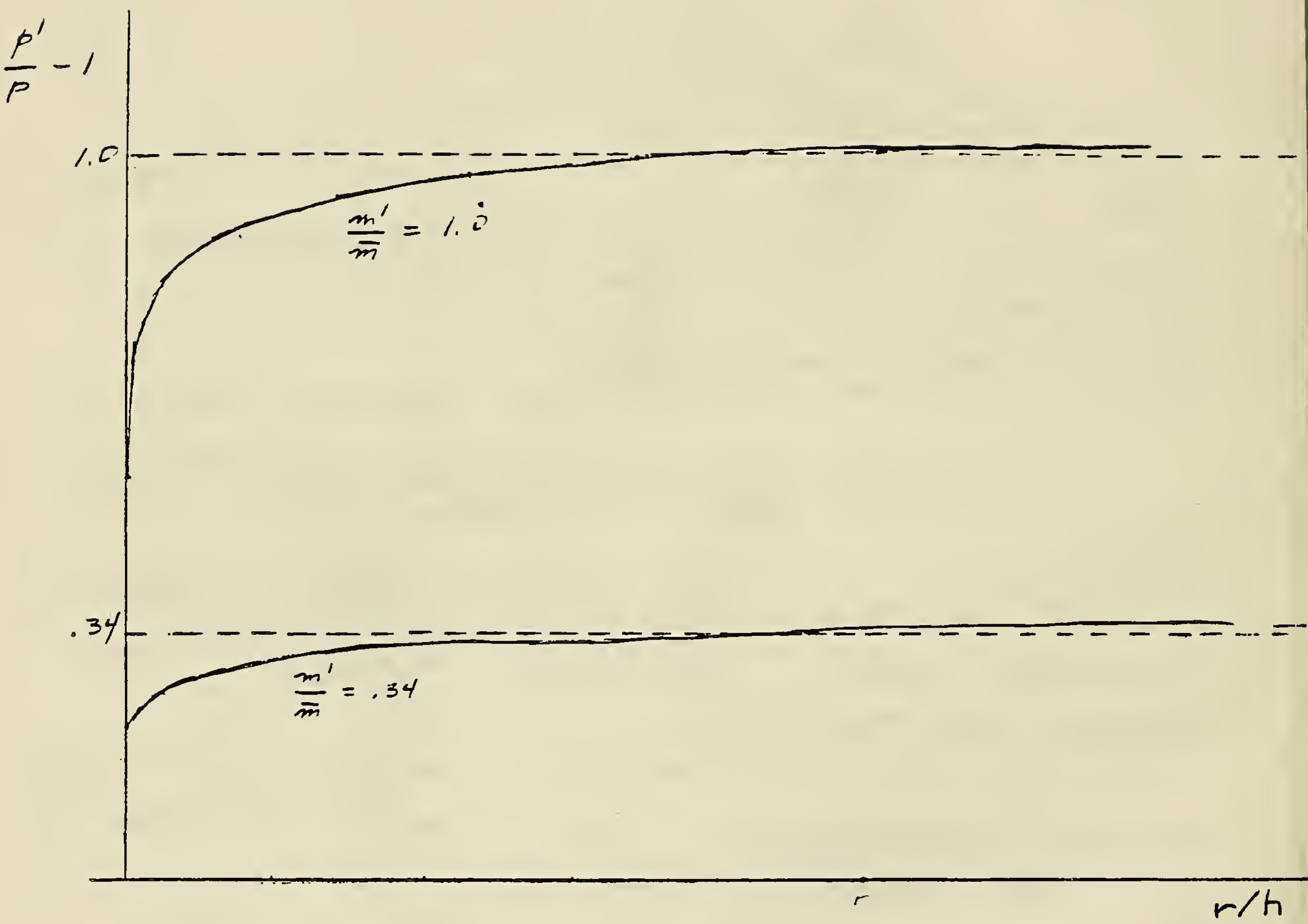

Figure 7

Instantaneous post-infusion price in model described in text, as function of ratio of capitalists' time horizon to speed of search in retail market, for two different values of monetary infusion. Note the singular price behavior in Walrasian limit when speed of search becomes infinite.

Proof. Combine (6.2) and (6.7) to obtain

$$\frac{w'}{w} = \left(\frac{p'}{p}\right)\left(\frac{1-F(pt)}{1-F(p)}\right), \tag{6.8}$$

where

$$\frac{1-F(pt)}{1-F(p)} = \frac{1-e^{zt} - z + zt}{1-e^{z}} \equiv B(t;z). \tag{6.9}$$

From (6.8) and the fact that $0\leq t\leq 1$ we have $w'/w\geq p'/p\geq 1$, Q.E.D.

This argument implies the

Corollary The instantaneous post-infusion real wage w'/p' exceeds the pre-infusion level w/p for all feasible (m',z).

We may now inquire whether the instantaneous price and wage levels are above or below their asymptotic levels $p^* = (\bar{m}+m')p/\bar{m}$ and $w^* = (\bar{m}+m')w/\bar{m}$. The signs of $p'-p^*$ and $w'-w^*$ each depend on the ratio of the discount rate to the rate of search, r/h, and the ratio, $m'/\bar{m}$, of the infusion to the pre-infusion mean. It will be shown that with short time horizons and sufficiently large infusions, price is larger than its asymptotic level. The converse is true for regimes in which capitalists' time horizons are long compared to the mean time between retail transactions; the converse also is true when the infusion is small. Formally, we have:

Theorem 6.4. (a) If the elasticity z is less than -2, the instantaneous post-infusion price always lies below its asymptotic level, independent of the size of the monetary infusion. (b) If the elasticity z lies in the interval (-2,-1), the instantaneous post-infusion price overshoots its asymptotic level if the ratio $m'/\bar{m}$ is sufficiently large.

Proof. (a) Consider the asymptotic quantity $t^*=(p^*-m')/p$, which may be expressed as

$$t^* = 1 + \frac{m'}{\bar{m}} \; [\tfrac{1}{2}+1/z] \tag{6.10}$$

by virtue of (4.6). Since $t-t^*=(p'-p^*)/p$, $t-t^*$ serves as a surrogate for $p'-p^*$ in our investigation of sign. We observe that for values of z less than -2, t^* is greater than 1, and a fortiori greater than all feasible values of t. It follows that p' is less than p^*.

(b). For $m'/\bar{m}$ sufficiently large, $t^*\leq 0$ and therefore $t\geq t^*$. Q.E.D.

To find the boundary of the overshoot region, subtract (6.10) from (6.7), use (4.6), and rearrange terms. Defining $M = m'/\bar{m}$,

$$t-t^* = A(t;z) - 1 - M. \tag{6.11}$$

Equation (6.11) tells us that the boundary of the overshoot region is described by a curve M(z) that satisfies

$$A(t^*(M,z);z) = 1+M, \tag{6.12}$$

where $t^*(M,z)$ is given by (6.10). In Figure 8 we show the results of a numerical calculation of M(z).

We close with remarks concerning the relationship between the instantaneous post-infusion wage, w', and its asymptotic value, w^*. From (6.7)-(6.9) and the proportionality of the asymptotic wage to the money supply, we have

$$\frac{w'}{w^*} = A(t;z)B(t;z)(1+M)^{-1}. \tag{6.13}$$

Using (4.6) and (6.7) to solve for M, we can write (6.13) as

$$\frac{w'}{w^*} = A(t;z)B(t;z) \left(1 + \frac{A(t;z)-t}{\frac{1}{2} - 1/z}\right)^{-1}. \tag{6.14}$$

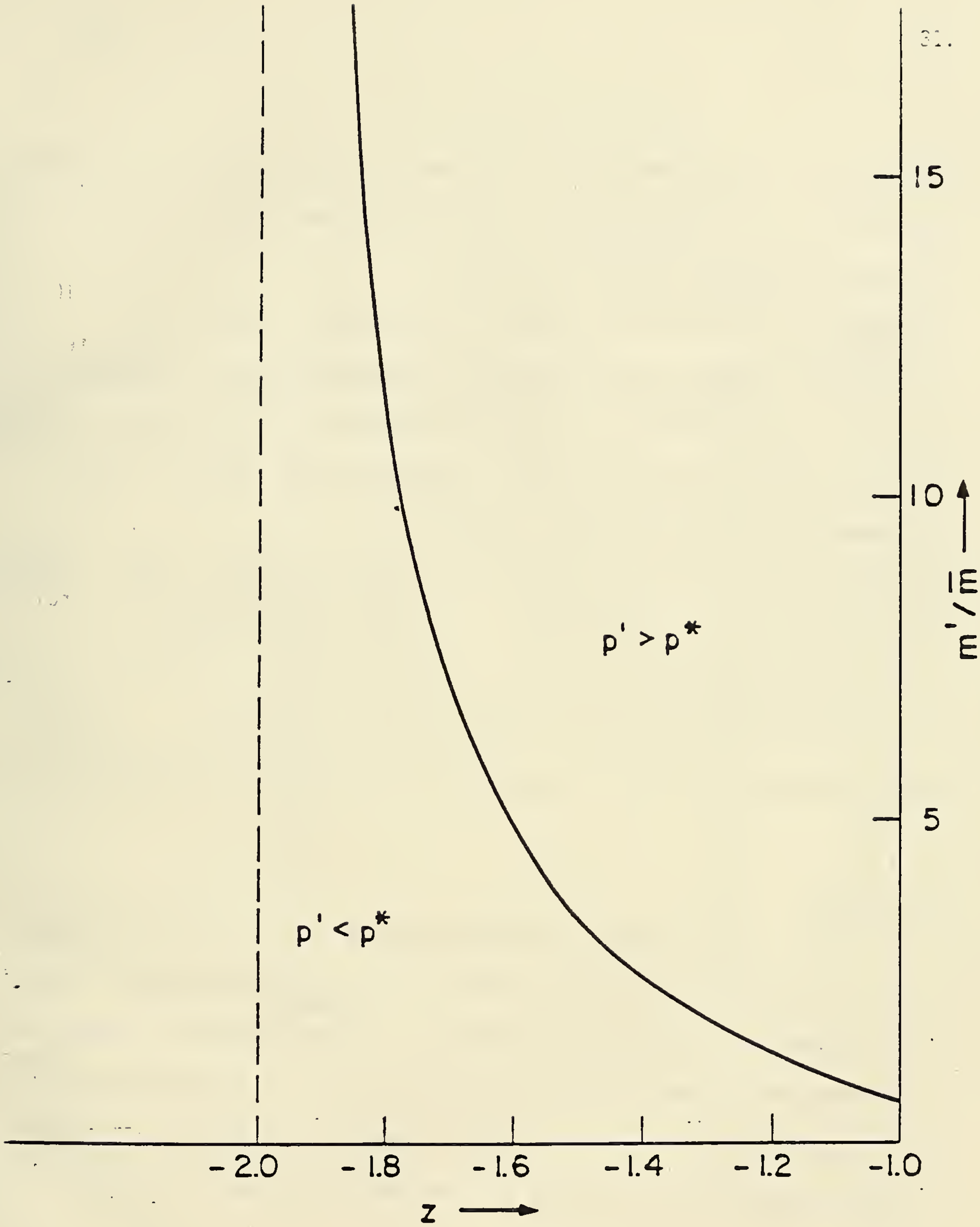

Figure 8

Overshoot region for instantaneous post-infusion retail price, as dependent on aggregate demand elasticity, z, and relative infusion $m'/\bar{m}$. As shown in text, to left of line z=-2, overshoot does not occur independent of the size of the infusion.

In Figure 9 we show the results of a numerical calculation of the boundary curve $w'/w^* = 1$ in (z,M) space. Analytically examining (6.14) for $-z$ very large and also very close to 1, we have

Theorem 6.5 (a) If the elasticity z is sufficiently small, the instantaneous wage always lies below its asymptotic level, independent of the size of the infusion. (b) If the elasticity z is sufficiently close to -1, the instantaneous post-infusion wage exceeds its asymptotic level, independent of the size of the infusion.

Proof: (a) From (6.7), we see that as z decreases without limit, t goes to one. From (6.14) it follows that w'/w^* goes to $(1+M/2)/(1+M)$.

(b) As z approaches -1, w'/w^* approaches

$$(2-e^{-t}-t)^2\,(1-e^{-1})^{-1}(1-e^{-t}+(2/3)(2-e^{-t}-2t+te^{-t}))^{-1} \tag{6.15}$$

The expression in (6.15) is decreasing in t and equals one at $t=1$. Q.E.D.

Conclusion

We see three directions for future research. (1) We plan to change the labor market to incorporate search and long term contracts. (2) We will attempt to introduce an interest rate to the stochastic process that describes the accumulation of purchasing power. (3) Success in step (2) will permit us to add a second asset. Steps (2) and (3) should enable us to explore many of the features of a real monetary system.

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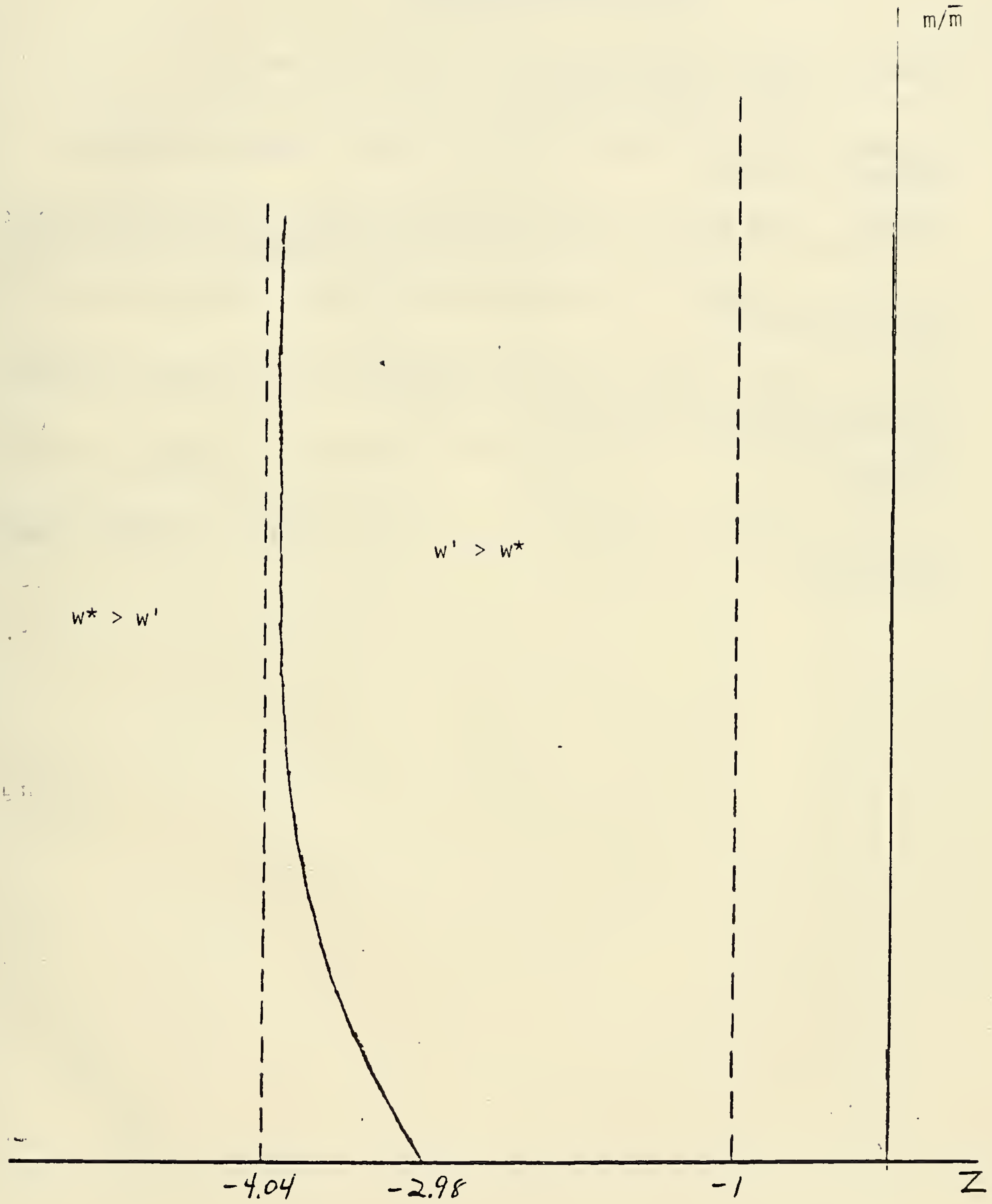

Figure 9

Overshoot region for instantaneous post-infusion wage, as dependent on aggregate demand elasticity, z, and relative infusion m'/$\bar{m}$.

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